Вариант 22

№1.

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{\ln(n+1)} \left(x - \frac{2}{3}\right)^n$$

$$\lim_{n\to\infty} \frac{3^{n+1} \left|x-\frac{2}{3}\right|^{n+1} \cdot \cancel{\ln(n+1)}}{\cancel{\ln(n+2)} \cdot \cancel{3^n} \cdot \cancel{\left|x-\frac{2}{3}\right|^n}} = \lim_{n\to\infty} 3\left|x-\frac{2}{3}\right| < 1$$

$$x \in \left(\frac{1}{3}; 1\right)$$

$\exists\, x = \frac{1}{3}$:

$$\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)} \geq \sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow \text{ряд расходится}$$

$\exists\, x = 1$. Проверим на усл. сх-ть:

$$\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0. \quad |a_n| > |a_{n+1}| \Rightarrow \text{усл. сх-ть.}$$

Ответ: $\left(\frac{1}{3}; 1\right]$ – обл-ть сх-ти

№2.

$$f(x) = \frac{1}{x(x-1)}$$ по степеням $(x+2)$

$t = x+2$; $x = t-2$

$$\frac{1}{(t-2)(t-3)} = \frac{A}{t-2} + \frac{B}{t-3} = \frac{At-3A+Bt-2B}{(t-2)(t-3)} =$$

$$\begin{cases} A+B=0 \\ -3A-2B=1 \end{cases} \quad \begin{cases} A=-1 \\ B=1 \end{cases} = \frac{1}{2-t} - \frac{1}{3-t}$$

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \dots$$

$$\frac{1}{2-t} = \frac{1}{2(1-\frac{t}{2})} = \frac{1}{2}\left(1 + \frac{t}{2} + \frac{t^2}{4} + \dots + \frac{t^n}{2^n} + \dots\right) =$$

$$= \frac{1}{2} + \frac{t}{4} + \frac{t^2}{8} + \dots + \frac{t^n}{2^{n+1}} + \dots = \sum_{n=0}^{\infty} \frac{t^n}{2^{n+1}}$$

$$\frac{1}{(3-t)} = \frac{1}{3}\left(1-\frac{t}{3}\right)^{-1} = \frac{1}{3}\left(1 + \frac{t}{3} + \frac{t^2}{9} + \dots + \frac{t^n}{3^n} + \dots\right) =$$

$$= \frac{1}{3} + \frac{t}{9} + \frac{t^2}{27} + \dots + \frac{t^n}{3^{n+1}} + \dots = \sum_{n=0}^{\infty} \frac{t^n}{3^{n+1}}$$

Исходный ряд: $$\sum_{n=0}^{\infty} \frac{t^n(3^{n+1}-2^{n+1})}{6^{n+1}} = \sum_{n=0}^{\infty} \frac{(x+2)^n(3^{n+1}-2^{n+1})}{6^{n+1}}$$

$$-1 < -\left(\frac{x+2}{2}\right) < 1, \text{ т.к. } m=-1$$

$$x \in (-4; 0)$$

№3

$$y' = 2x - \ln y + 3 \qquad y(-3) = 1$$

$$y'(-3) = -6 + 3 = -3$$

$$y'' = 2 - \frac{y'}{y} \qquad y''(-3) = 2 + 3 = 5$$

$$y''' = \frac{(y')^2 - y''\cdot y}{y^2} \qquad y'''(-3) = \frac{9 - 5}{1} = 4$$

$$y^{IV} = \frac{\left(2y'\cdot y'' - y\cdot y''' - y'\cdot y''\right)y^2 - 2yy'\left((y')^2 - y\cdot y''\right)}{y^4} =$$

$$= \frac{2y^{\cancel{2}}y'y'' - y^{\cancel{3}\,2}\cdot y''' - y^{\cancel{2}}y'\cdot y'' - 2\cancel{y}(y')^3 + 2y^{\cancel{2}}y'y''}{y^{\cancel{4}\,3}} =$$

$$= \frac{3yy'y'' - y^2\cdot y''' - 2(y')^3}{y^3}$$

$$y^{IV}(-3) = 3(-3)\cdot 5 - 4 + 2\cdot 27 = -9\cdot 5 - 4 + 54 = 5$$

$$y \simeq 1 - 3(x+3) + \frac{5(x+3)^2}{2!} + \frac{4(x+3)^3}{3!} + \frac{5(x+3)^4}{4!}$$

№4.

См. вариант 1.

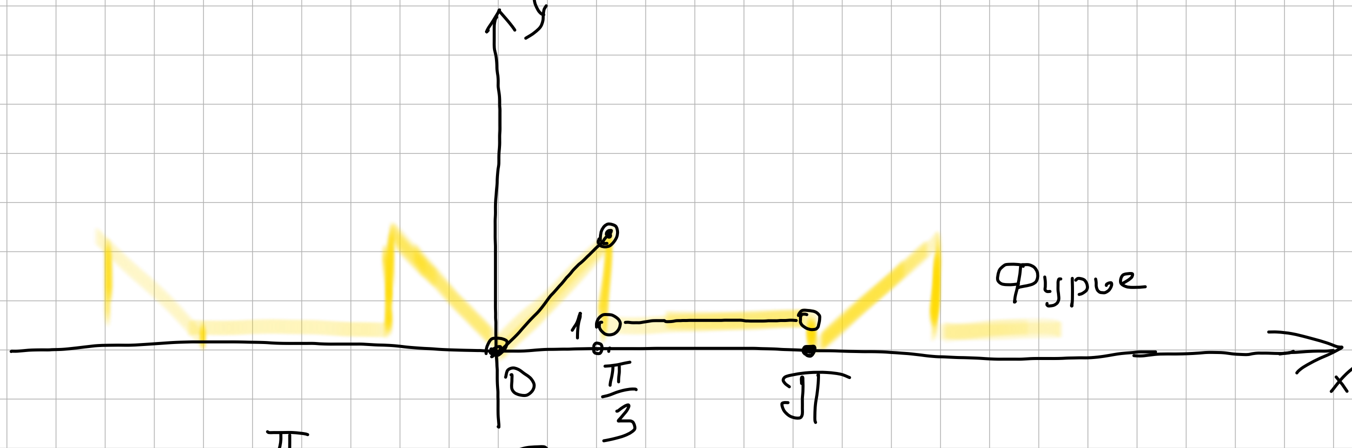
№ 5.

См. вариант 13

№6

Разложить по cos:

$$f(x)=\begin{cases}2x & \text{при } 0<x<\frac{\pi}{3}\\ \pi-3 & \text{при } \frac{\pi}{3}<x<\pi\end{cases}$$



$$a_0=\frac{2}{\pi}\left(\int_0^{\frac{\pi}{3}}2x\,dx+\int_{\frac{\pi}{3}}^{\pi}(\pi-3)\,dx\right)=\frac{2}{\pi}\left(\frac{\pi^2}{9}\cdot\frac{2\pi(\pi-3)}{3}\right)=$$

$$=\frac{4\pi^2(\pi-3)}{27}$$

$$a_n=\frac{2}{\pi}\left(2\int_0^{\frac{\pi}{3}}x\cdot\cos(nx)\ \overset{①}{}+(\pi-3)\int_{\frac{\pi}{3}}^{\pi}\cos(nx)\ \overset{②}{}\right)=$$

$$① \left|\begin{array}{ll} v=x & u=\sin(nx)\cdot\frac{1}{n}\\ v'=1 & u'=\cos(nx)\end{array}\right|=\frac{2x}{n}\sin(nx)\Big|_0^{\frac{\pi}{3}}-$$

$$-2\int_0^{\frac{\pi}{3}}\sin(nx)\frac{1}{n}dx=\frac{2\pi}{3n}\sin\left(\frac{\pi n}{3}\right)+2\cos(nx)\frac{1}{n^2}\Big|_0^{\frac{\pi}{3}}=$$

$$=\frac{2\pi}{3n}\sin\left(\frac{\pi n}{3}\right)+\frac{2}{n^2}\cos\left(\frac{\pi n}{3}\right)-\frac{2}{n^2}$$

② $\frac{(\pi-3)}{n}\sin(nx)\Big|_{\frac{\pi}{3}}^{\pi} = \frac{(3-\pi)}{n}\sin\left(\frac{\pi n}{3}\right)$

$= \frac{2}{\pi}\left(\cancel{\frac{2\pi}{3n}\sin\left(\frac{\pi n}{3}\right)} + \frac{2}{n^2}\cos\left(\frac{\pi n}{3}\right) - \frac{2}{n^2} + \frac{9}{3n}\sin\left(\frac{\pi n}{3}\right) -\right.$

$\left.- \frac{\cancel{3}\pi}{3n}\sin\left(\frac{\pi n}{3}\right)\right)\Bigg) = \frac{2}{\pi}\left(\frac{2}{n^2}\cos\left(\frac{\pi n}{3}\right) - \frac{2}{n^2} +\right.$

$\left.+ \frac{9-\pi}{3n}\sin\frac{\pi n}{3}\right)$

$b_n = 0$. Итого:

$$f(x) \simeq \frac{2\pi^2(\pi-3)}{27} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{2}{n^2}\cos\left(\frac{\pi n}{3}\right) - \frac{2}{n^2} + \frac{9-\pi}{3n}\sin\left(\frac{\pi n}{3}\right)\right) \cdot$$

$\cdot \cos(nx)$