Вариант 5

№ 1

$$\sum_{n=1}^{\infty}\left(-\frac{n+1}{2n+3}\right)^n (x+3)^n$$

См. вариант 15

№2

$$f(x) = \cos\frac{x}{4} \quad \text{по} \quad x - \pi = t \ ; \quad x = t + \pi$$

$$\cos\left(\frac{t+\pi}{4}\right) = \cos\left(\frac{t}{4}\right)\frac{\sqrt{2}}{2} - \sin\left(\frac{t}{4}\right)\frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{2}}{2}\left(\sum_{n=0}^{\infty}(-1)^n\left(\frac{t^{2n}}{(2n)!} + \frac{t^{2n-1}}{(2n-1)!}\right)\right) =$$

$$\frac{\sqrt{2}}{2}\left(\sum_{n=0}^{\infty}(-1)^n\left(\frac{(x-\pi)^{2n}}{(2n)!} + \frac{(x-\pi)^{2n-1}}{(2n-1)!}\right)\right)$$

$$t = x - \pi \in \mathbb{R}$$

№3

$$\int_0^{0.4} \frac{\ln\left(1+\frac{x}{2}\right)dx}{x} \qquad \text{С точностью до } 0.001$$

$$\frac{\ln\left(1+\frac{x}{2}\right)}{x} = \frac{1}{x}\left(\frac{x}{2} - \frac{x^2}{2^3} + \frac{x^3}{2^3\cdot 3} - \ldots\right) =$$

$$= \frac{1}{2} - \frac{x}{8} + \frac{x^2}{24} - \ldots$$

$$\int_0^{0.4} \frac{1}{2}dx = \frac{2}{10} = \frac{1}{5}; \quad \int_0^{0.4} -\frac{x}{8}dx = -\frac{x^2}{16}\Bigg|_0^{0,4} = \frac{-1}{100}$$

$$\int_0^{0.4} \frac{x^2}{24}dx = \frac{x^3}{72}\Bigg|^{0.4} = \frac{0,064}{72} = \frac{0,008}{9} < \frac{1}{1000}$$

$$\int_0^{0.4} \frac{\ln\left(1+\frac{x}{2}\right)}{x}dx = \frac{20}{100} - \frac{1}{100} = \frac{19}{100}$$

№ 5. (см вар. 10)

$$I(y) = \int_1^{\sqrt[3]{y}} \frac{x^2}{x^3+1} e^{y\sqrt{x^3+1}}\,dx$$

$$I'(y) = \frac{\cancel{y^{\frac{2}{3}}} \cdot e^{y\sqrt{y+1}}}{3\cdot(y+1)\,\cancel{y^{\frac{2}{3}}}} + \overset{(*)}{\int_1^{\sqrt[3]{y}}} \frac{x^2}{\sqrt{x^3+1}} e^{y\sqrt{x^3+1}}\,dx \ominus$$

$$(*) \int_1^{\sqrt[3]{y}} \frac{x^2}{\sqrt{x^3+1}} e^{y\sqrt{x^3+1}}\,dx = \left| \begin{array}{l} t = x^3+1 \\ dt = 3x^2\,dx \\ dx = \frac{dt}{3x^2} \end{array} \right| =$$

$$= \frac{1}{3}\int_2^{y+1} \frac{e^{y\sqrt{t}}}{\sqrt{t}}\,dt = \frac{2}{3y} e^{y\sqrt{t}}\Big|_2^{y+1} = \frac{2}{3y}\left(e^{y\sqrt{y+1}} - e^{y\sqrt{2}}\right)$$

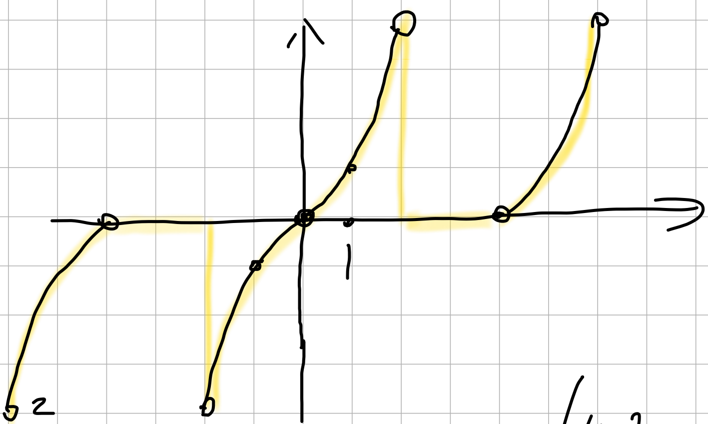
$$\ominus \frac{e^{y\sqrt{y+1}}}{3(y+1)} + \frac{2}{3y}\left(e^{y\sqrt{y+1}} - e^{y\sqrt{2}}\right)$$

№6

$$f(x)=\begin{cases} x^2 & \text{при } 0<x<2 \\ 0 & \text{при } 2\leq x<4 \end{cases} \quad \text{по } \sin$$

$a_0=0$

$a_n=0$



$$b_n=\frac{1}{2}\int_0^2 x^2\cdot\sin\left(\frac{\pi n x}{4}\right)dx=\frac{1}{2}\left(\frac{4x^2}{\pi n}\cos\left(\frac{\pi n x}{4}\right)-\frac{32x}{\pi^2 n^2}\cos\left(\frac{\pi n x}{4}\right)-\frac{128}{\pi^3 n^3}\cos\left(\frac{\pi n x}{4}\right)\right)\Bigg|_0^2=\frac{1}{2}\cdot\Bigg($$

D	I
$+\ x^2$	$\sin\frac{\pi n x}{4}$
$-\ 2x$	$-\frac{4}{\pi n}\cos \sin x$
$+\ 2$	$-\frac{16}{\pi^2 n^2}\sin\frac{\pi n x}{4}$
$-\ 0$	$\frac{64}{\pi^3 n^3}\cos\frac{\pi n x}{4}$

$$\frac{16}{\pi n}\cos\left(\frac{\pi n}{2}\right)-\frac{64}{\pi^2 n^2}\cos\left(\frac{\pi n}{2}\right)-\frac{128}{\pi^3 n^3}\cos\left(\frac{\pi n}{2}\right)+\frac{128}{\pi^3 n^3}\Bigg)=$$

$$=\cos\left(\frac{\pi n}{2}\right)\left(\frac{8}{\pi n}-\frac{32}{\pi^2 n^2}-\frac{64}{\pi^3 n^3}\right)+\frac{64}{\pi^3 n^3}$$

$S(0)=S(4)=0$

$$S(x)=\sum_{n=1}^{\infty}\left(\cos\left(\frac{\pi n}{2}\right)\left(\frac{8}{\pi n}-\frac{32}{\pi^2 n^2}-\frac{64}{\pi^3 n^3}\right)+\frac{64}{\pi^3 n^3}\right)\cdot\sin\left(\frac{\pi n x}{4}\right)$$

$S(-4)=0$

$S(4)=0$